

# Airplane Dynamics, Modeling, and Control

Dr. Eugene A. Morelli

NASA Langley Research Center

May 14, 1997

# Overview

---

- | General Airplane Dynamics
- | Modeling for Control Design
- | Control Design for Airplanes
- | Demonstrations

# Airplane Dynamics

---

- | The Airplane is a Nonlinear Dynamical System
- | Newton's 2nd Law for a Rigid Body

- *Translational Motion :*  $\dot{\vec{p}} = \vec{F}$

- *Rotational Motion :*  $\dot{\vec{h}} = \vec{M}$

# Assumptions

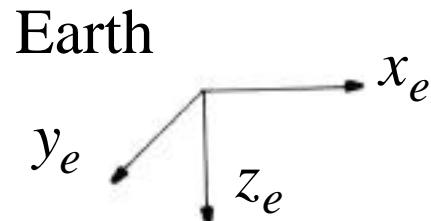
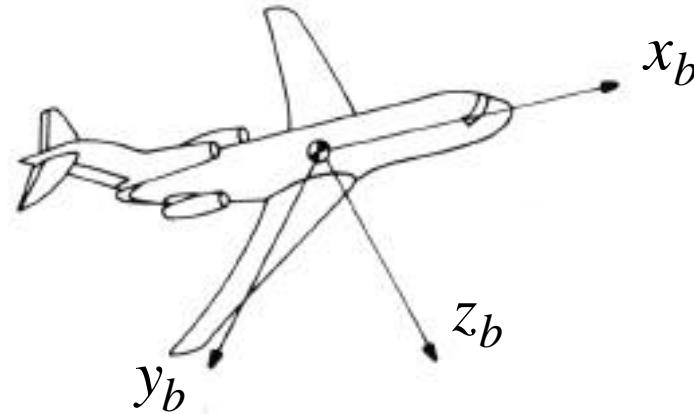
---

- | Earth is an inertial reference, no curvature
- | Airplane is a rigid body with lateral symmetry
- | Thrust acts along fuselage through the c.g.
- | Still atmosphere (no winds, no gusts)
- | Constant mass, no internal mass movements

# Axis Systems

---

- | Equations written in body axes
  - Fixed to the airplane, constant inertia
  - Rotating axes  $\rightarrow$  nonlinear inertial terms



# Nonlinear Equations of Motion

---

- Translational motion of the c.g.

$$m\dot{\vec{V}} + \vec{\omega} \times m\vec{V} = \vec{F}_{aero} + \vec{F}_{prop} + \vec{F}_{gravity}$$

- Rotational motion about the c.g.

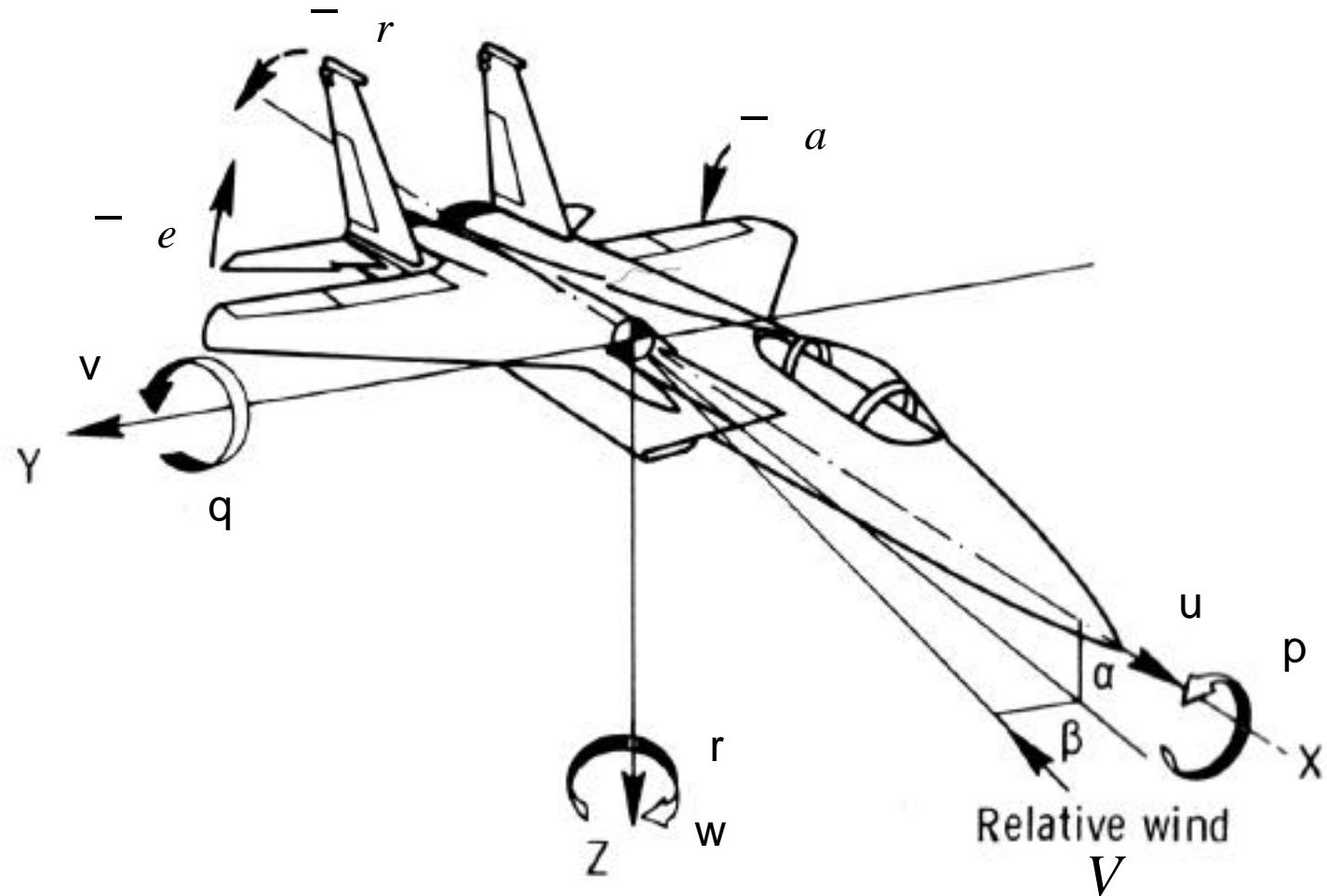
$$\dot{\vec{I}} + \vec{\omega} \times \vec{I} = \vec{M}_{aero}$$

- Rotational kinematics

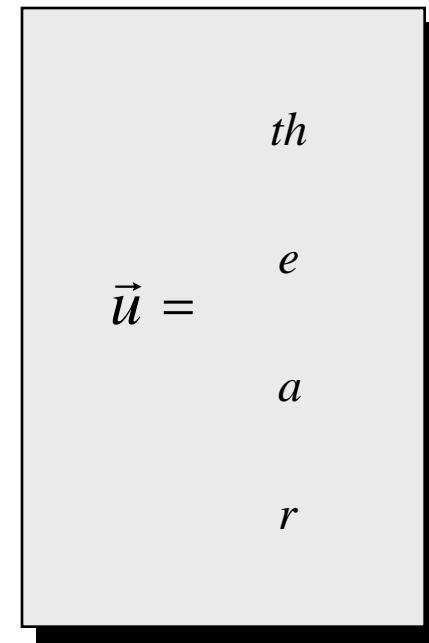
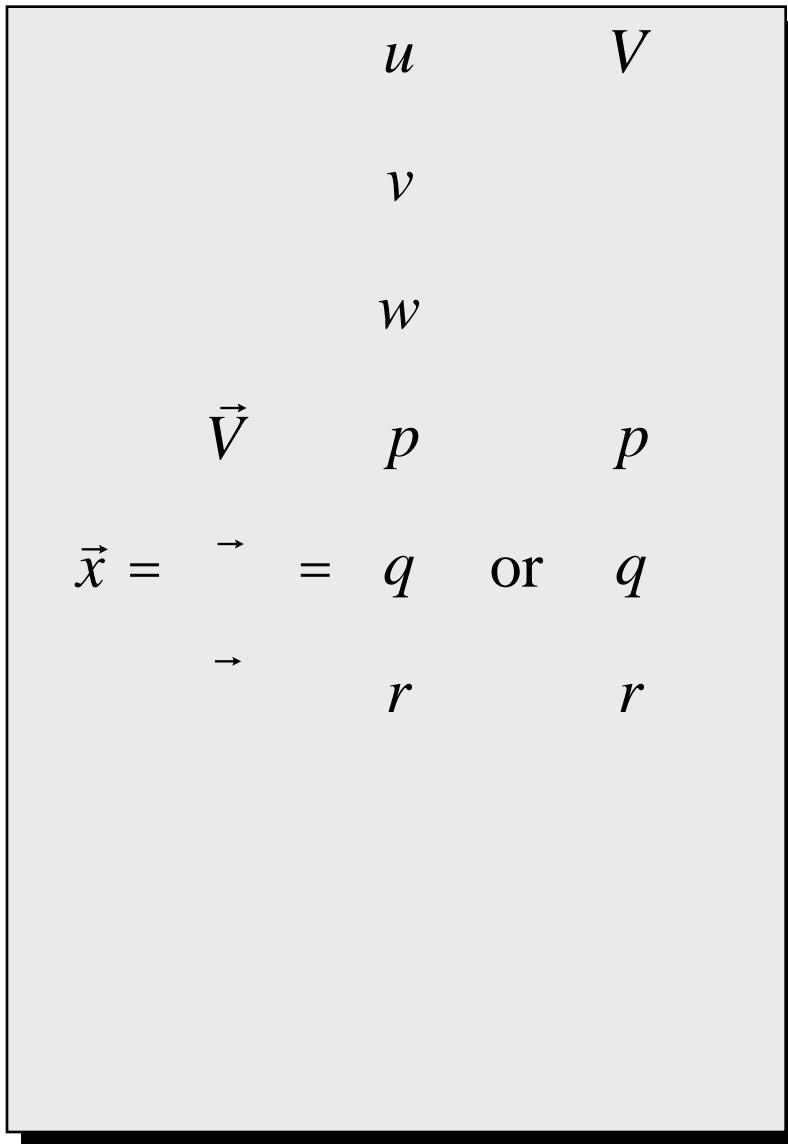
$$\dot{\vec{\omega}} = \vec{L}$$

# States and Controls

---



# States and Controls



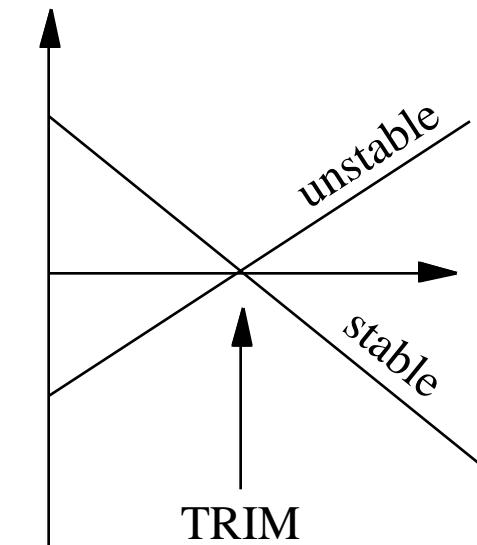
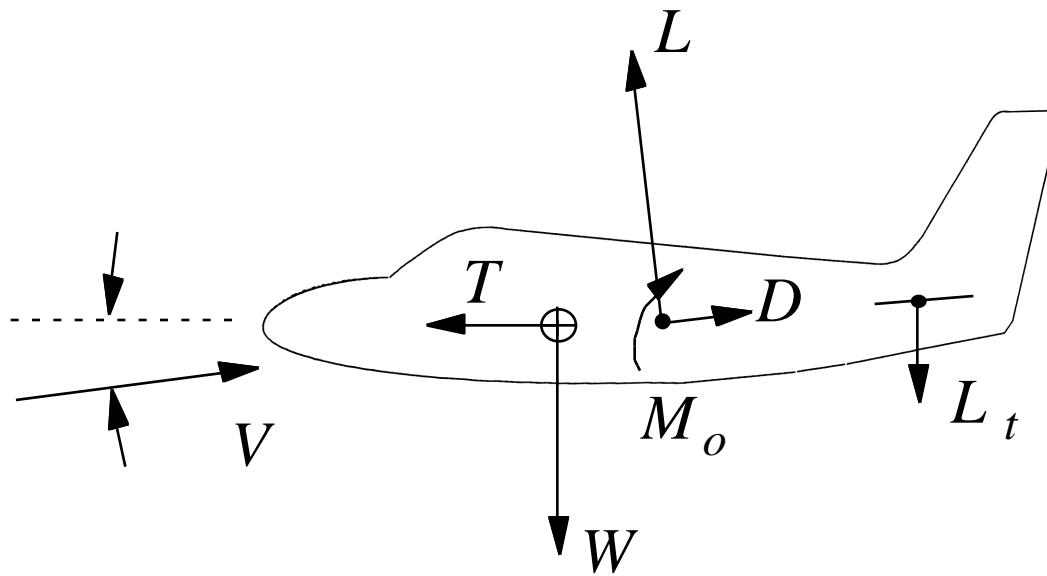
# Steady Flight

| Nonlinear Equations of Motion :

$$\dot{\vec{x}} = \vec{f}(\vec{x}, \vec{u})$$

| Define steady state (trim) :

$$\vec{0} = \vec{f}(\vec{x}_o, \vec{u}_o)$$



# Modeling for Control Design

---

| Define steady state (trim) :  $\vec{0} = \vec{f}(\vec{x}_o, \vec{u}_o)$

● Linearize about trim :

$$\dot{\vec{x}} = [\vec{f}/\vec{x}]_{\vec{x}_o, \vec{u}_o} \vec{x} + [\vec{f}/\vec{u}]_{\vec{x}_o, \vec{u}_o} \vec{u}$$

● Linear model for analysis and control design :

$$\dot{\vec{x}} = A \vec{x} + B \vec{u}$$

# Linear Models

---

- Linearized model variables are perturbations
- Linearization naturally decouples dynamics

Longitudinal

$$V$$
$$\vec{x} = q \quad \vec{u} = e^{\text{th}}$$

Lateral / Directional

$$p$$
$$\vec{x} = r \quad \vec{u} = \begin{matrix} a \\ r \end{matrix}$$

# Longitudinal Linear Equations

$$\dot{V} = (X_V + T_V)V + X_{th} - g + T_{th} + X_{e_e}$$

$$\dot{\cdot} = Z_V V + Z_{e_e} + q + Z_{e_e}$$

$$\dot{q} = M_V V + M_{q_e} + M_q q + M_{e_e}$$

$$\dot{q} = q$$

# Laplace Transform

$$s\tilde{V} = (X_V + T_V)\tilde{V} + X_{th}^{\sim} - g^{\sim} + T_{th}^{\sim} + X_e^{\sim}$$

$$s^{\sim} = Z_V \tilde{V} + Z_e^{\sim} + \tilde{q} + Z_e^{\sim}$$

$$s\tilde{q} = M_V \tilde{V} + M_e^{\sim} + M_q \tilde{q} + M_e^{\sim}$$

$$s^{\sim} = \tilde{q}$$

# Computing Transfer Functions

$$\begin{array}{ccccccc} s - (X_V + T_V) & -X & 0 & g & \tilde{V} & T_{th} & X_e \\ -Z_V & \boxed{s - Z & -1} & & 0 & \sim & 0 & \boxed{Z_e} & \tilde{th} \\ -M_V & \boxed{-M & s - M_q} & & 0 & \tilde{q} & 0 & \boxed{M_e} & \tilde{e} \\ 0 & 0 & -1 & s & \sim & 0 & 0 \end{array}$$

# Computing Transfer Functions

$$\frac{\tilde{z}}{\tilde{e}} = \frac{\begin{vmatrix} s - (X_V + T_V) & X_e & 0 & g \\ -Z_V & Z_e & -1 & 0 \\ -M_V & M_e & s - M_q & 0 \\ 0 & 0 & -1 & s \end{vmatrix}}{\begin{vmatrix} s - (X_V + T_V) & -X & 0 & g \\ -Z_V & s - Z & -1 & 0 \\ -M_V & -M & s - M_q & 0 \\ 0 & 0 & -1 & s \end{vmatrix}}$$

# Modeling Example

---

Airplane : F-16      c.g. position : 0.2  $\bar{c}$  (fwd)

Flight Condition : 5° AOA    10,000 ft    350 kts

Full Linear Model

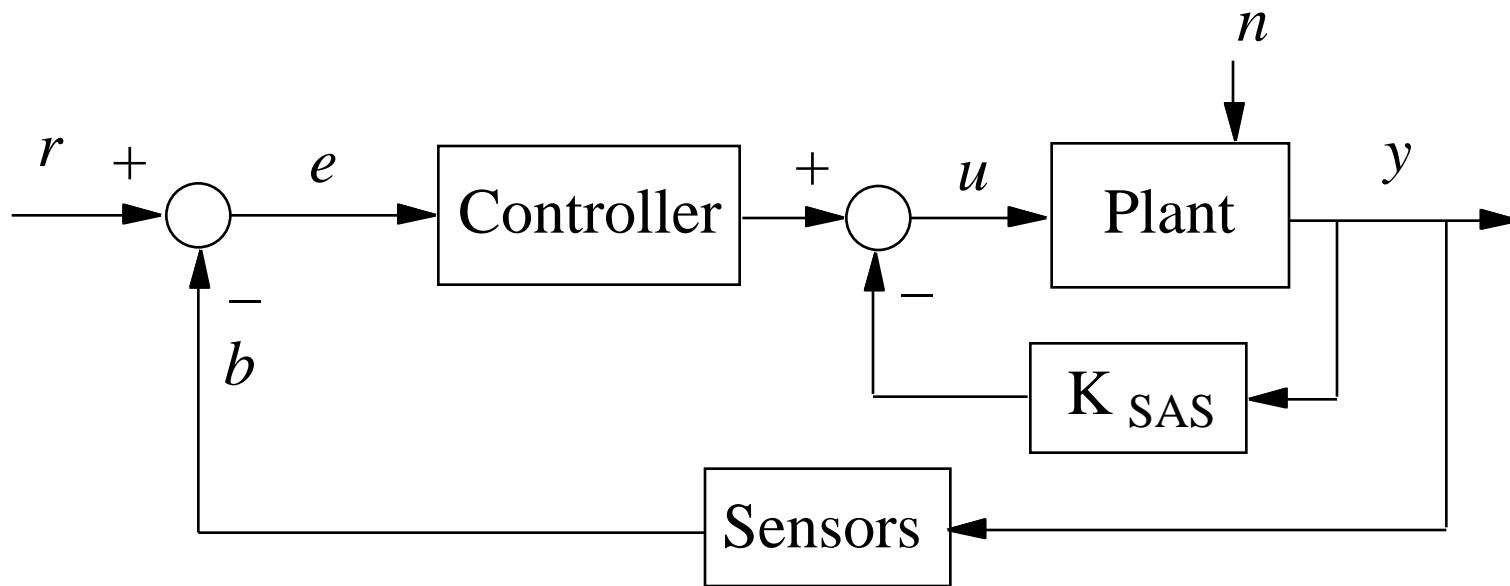
$$\frac{\tilde{e}}{\tilde{e}} = \frac{-0.19 \left( [s + 0.008]^2 + 0.08^2 \right)}{\left( [s + 0.008]^2 + 0.07^2 \right) \left( [s + 1.3]^2 + 2.9^2 \right)}$$

Short Period Approx.

$$\frac{\tilde{e}}{\tilde{e}} = \frac{-0.19}{\left( [s + 1.3]^2 + 2.9^2 \right)}$$

# Why Feedback Control?

- | Modify plant dynamics
- | Accurate regulation or tracking
- | Overcome plant uncertainty



# Airplane Control Tasks

---

- | Stability Augmentation System (SAS)
- | Control Augmentation System (CAS)
  - » pitch rate command system
  - » g-load command system
- | Autopilots (pilot relief)
  - » airspeed hold
  - » altitude hold
  - » heading hold
  - » turn coordination

# Choosing Feedback Quantity

---

## Stability Augmentation

---

$$\dot{q} = M + M_q q + M_e \left( e_{SAS} + e_{PILOT} \right)$$

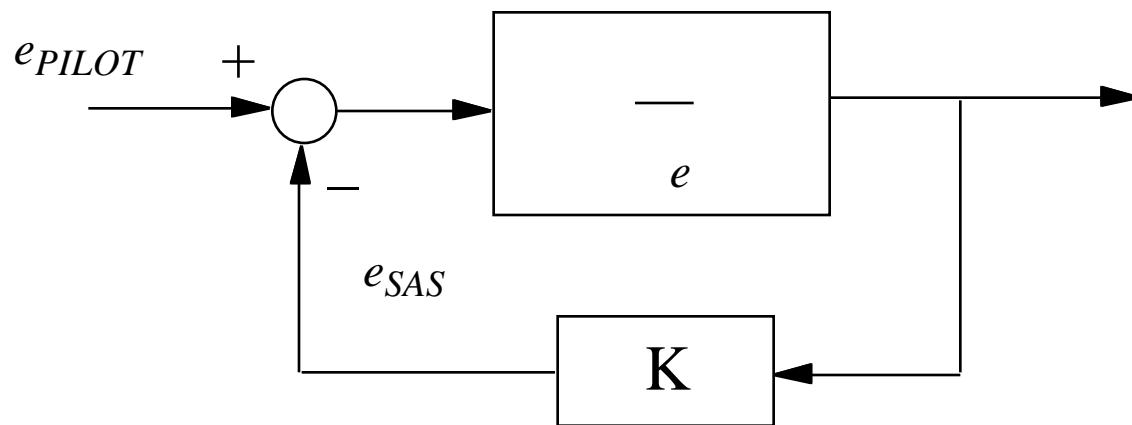
$$e_{SAS} = K$$

$$\dot{q} = \underbrace{\left( M + K M_e \right)}_{\text{effective } M} + M_q q + M_e e_{PILOT}$$

effective  $M$

# Stability Augmentation System (SAS)

---



# SAS Design Demonstration

---

Airplane : F-16      c.g. position : 0.2  $\bar{c}$  (fwd)

Flight Condition : 5° AOA    10,000 ft    350 kts

Short Period Approx.

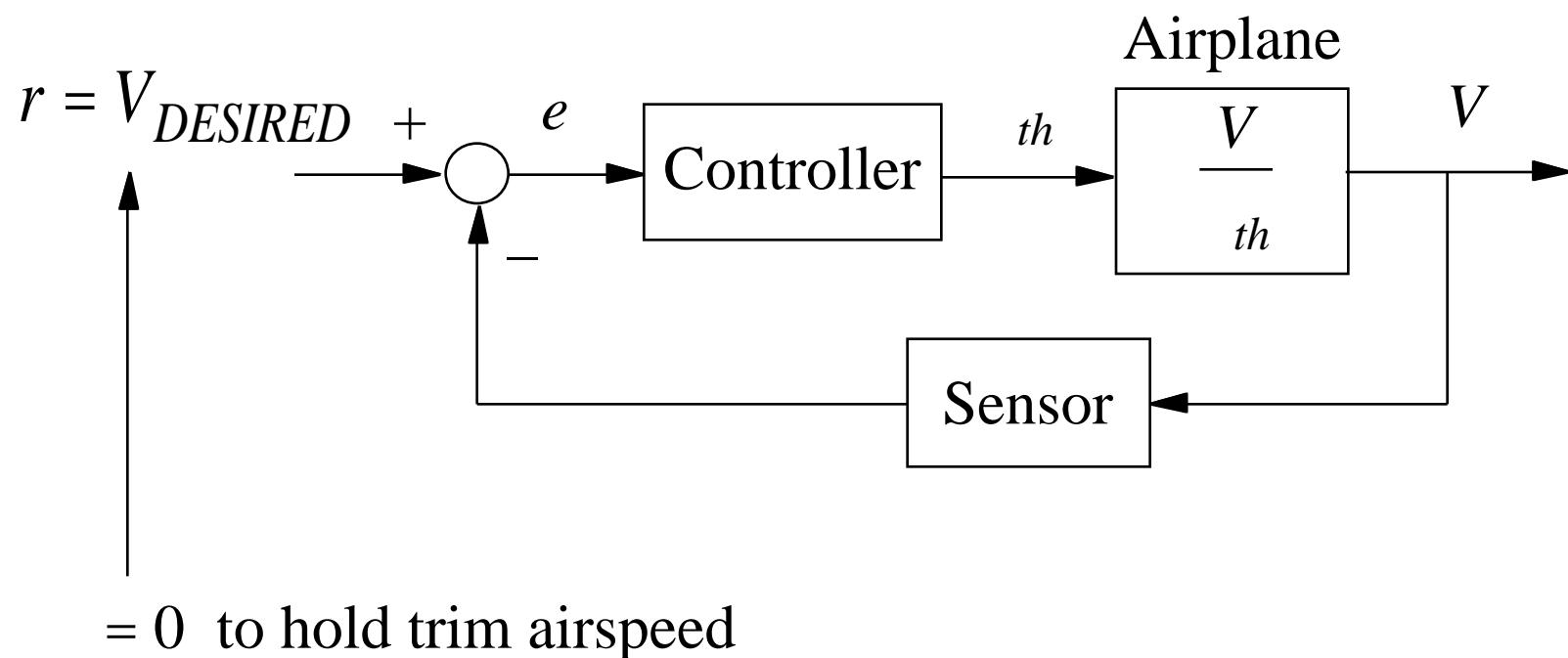
$$\frac{\tilde{\gamma}}{\tilde{e}} = \frac{-0.19}{([s + 1.3]^2 + 2.9^2)}$$

Full Linear Model      c.g. position : 0.35  $\bar{c}$  (nom)

$$\frac{\tilde{\gamma}}{\tilde{e}} = \frac{-0.18 ([s + 0.007]^2 + 0.08^2)}{([s + 0.08]^2 + 0.13^2)(s + 1.8)(s - 0.1)}$$

# Choosing Feedback Quantity

## Regulation or Tracking



# Airspeed Hold Demonstration

---

Airplane : F-16      c.g. position : 0.2  $\bar{c}$  (fwd)

Flight Condition : 5° AOA    10,000 ft    350 kts

Full Linear Model

$$\frac{\tilde{V}}{\tilde{v}_{th}} = \frac{0.17 \left( [s + 1.3]^2 + 6.1^2 \right) (s + 0.8)}{\left( [s + 0.008]^2 + 0.07^2 \right) \left( [s + 1.3]^2 + 2.9^2 \right)}$$

# Control System Design

---

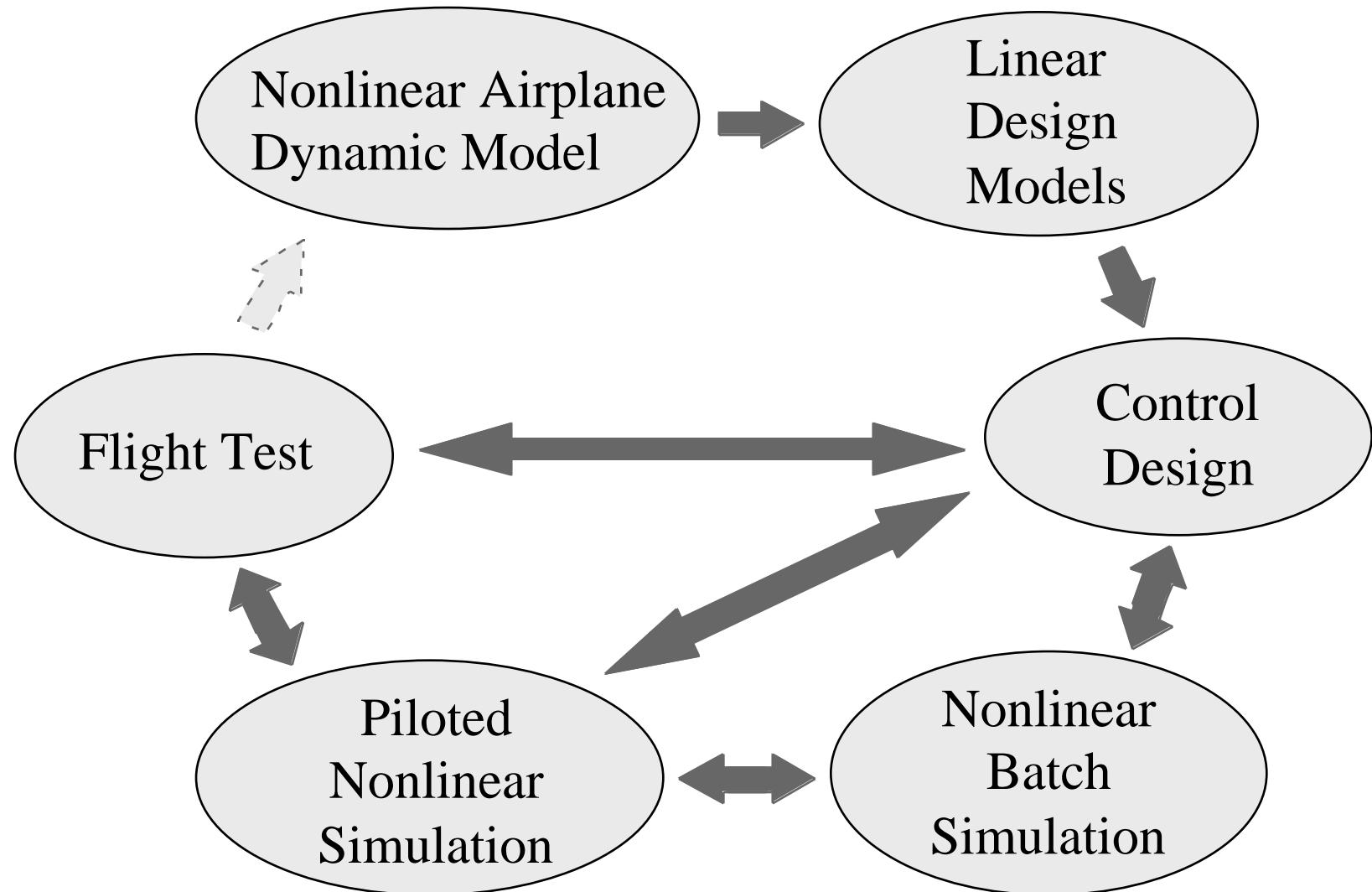
- | Close feedback control loops
  - » one at a time (classical control)
  - » many at once (modern control)
- | Use several linear models → design points
- | Link individual designs (gain scheduling)

# Practical Considerations

---

- | Control Effectiveness
  - » Deflection limits
  - » High AOA
  - » Nonlinearity
  - » Actuator Dynamics
- | Time delay
  - » Control surface rate limits
  - » Transport delay
- | Unmodeled effects
- | Pilot variability

# Control Design



# Summary

---

- | General Airplane Dynamics
- | Modeling for Control Design
- | Control Design for Airplanes
- | Demonstrations
- | References for Further Study